BVKT1 3. Schulaufgabe, nachgeholt am 18.06.13

1.1. $p_k(x) = \frac{1}{2}\left(x^2 - 4kx + (2k) - 4k^2\right) + 4$

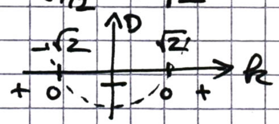
$= \frac{1}{2}\left((x-2k)^2 - 4k^2\right) + 4 = \frac{1}{2}(x-2k)^2 - 2k^2 + 4$

$\Rightarrow S(2k \,/\, 4 - 2k^2)$

1.2 $\frac{1}{2}x^2 - 2kx + 4 = 0 \;;\; D = 4k^2 - 4 \cdot \frac{1}{2} \cdot 4 = 4k^2 - 8$

$D = 0$, also $4k^2 - 8 = 0 \Leftrightarrow k^2 = 2 \Rightarrow k_{1/2} = \pm\sqrt{2}$

$D > 0$ für $k \in \mathbb{R} \setminus]-\sqrt{2}\,;\sqrt{2}[$



1.3.1 $h(2) = \frac{1}{4} \cdot 2^3 + \frac{1}{2} 2^2 - 5 \cdot 2 + 8 = 2$

$p_k(2) = 2 \Rightarrow \frac{1}{2} \cdot 2^2 - 4k + 4 = 2 \Leftrightarrow -4k = -4 \Leftrightarrow k = 1$

1.3.2 $h(x) = p_1(x): \frac{1}{4}x^3 + \frac{1}{2}x^2 - 5x + 8 = \frac{1}{2}x^2 - 2x + 4$

$\Leftrightarrow \frac{1}{4}x^3 - 3x + 4 = 0 \Leftrightarrow x^3 - 12x + 16 = 0 \;;\; x_0 = 2$

$(x^3 + 0x^2 - 12x + 16) : (x-2) = x^2 + 2x - 8$

$-(x^3 - 2x^2)$

$\quad + 2x^2$ $\qquad\qquad = (x+4)(x-2)$

$\quad -(+2x^2 - 4x)$

$\qquad - 8x + 16$ $\qquad p_1(-4) = \frac{1}{2} \cdot 16 - 2 \cdot 4 + 4 = 20$

$\qquad -(-8x + 16)$

$\qquad\quad - \; -$ $\qquad S_1(2/2) \;;\; S_2(-4/20)$

2.1 $P: a + b + c = 4{,}25$

$Q: 16a + 4b + c = 2$ $\qquad -75a - 15b = 11{,}25 \;|\cdot(-5)$

$15a + 3b = -2{,}25$ $\quad \mathrm{II}'$

$R: 256a + 16b + c = -52$ $\qquad 255a + 15b = -56{,}25$ $\quad \mathrm{III}'$

$-5 \cdot \mathrm{II}' + \mathrm{III}': \; 180a = -45 \Leftrightarrow a = -\frac{45}{180} = -\frac{1}{4}$

$\mathrm{II}': -\frac{15}{4} + 3b = -2{,}25 \Leftrightarrow 3b = \frac{3}{2} \Leftrightarrow b = \frac{1}{2}$

$\mathrm{I}: c = 4{,}25 - \frac{1}{2} + \frac{1}{4} = 4$

$f(x) = -\frac{1}{4}x^4 + \frac{1}{2}x^2 + 4$

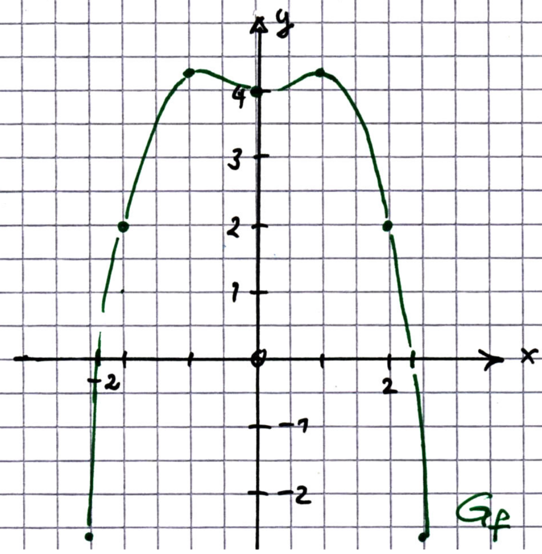
2.2 Sub: $-\frac{1}{4}u^2 + \frac{1}{2}u + 4 = 0 \;|\cdot(-4) \Leftrightarrow u^2 - 2u - 16 = 0$

$u_{1/2} = \frac{1}{2}\left(-2 \pm \sqrt{4 + 4 \cdot 16}\right) = -1 \pm \sqrt{17}$

$u_1 = -1 - \sqrt{17} < 0 \Rightarrow$ Resub ↯

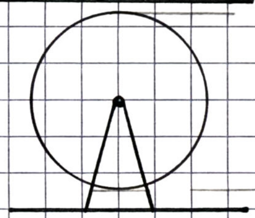
$u_2 = -1 + \sqrt{17} > 0 \Rightarrow x_{1/2} = \pm\sqrt{\sqrt{17} - 1}$ $\quad (\approx \pm 1{,}77)$

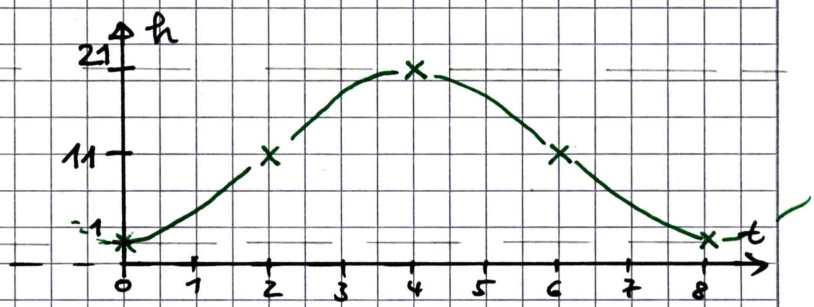
2.3



$W_f =]-\infty; 4{,}25]$

Aufgabe 3





$h(t) = -10 \cdot \cos\left(\frac{2\pi}{8} t\right) + 11 = -10 \cos\left(\frac{\pi}{4} t\right) + 11$

Aufgabe 4

$2 \cdot \sin\left(\frac{\pi}{2} x + \pi\right) = \sqrt{3} \Leftrightarrow \sin\left(\frac{\pi}{2} x + \pi\right) = \frac{1}{2}\sqrt{3}$

$\Rightarrow u_{1R} = \sin^{-1}\left(\frac{1}{2}\sqrt{3}\right) = \frac{1}{3}\pi$

$u_{1k} = \frac{1}{3}\pi + k \cdot 2\pi$

$\frac{\pi}{2} x_{1k} + \pi = \frac{1}{3}\pi + k \cdot 2\pi$

$x_{1k} = \left(\frac{1}{3}\pi - \pi + k \cdot 2\pi\right) \cdot \frac{2}{\pi} = \left(-\frac{2}{3}\pi + k \cdot 2\pi\right) \cdot \frac{2}{\pi}$

$\underline{x_{1k} = -\frac{4}{3} + 4k}$

$u_2 = \pi - u_{1R} = \pi - \frac{1}{3}\pi = \frac{2}{3}\pi$

$u_{2k} = \frac{2}{3}\pi + k \cdot 2\pi$

$\frac{\pi}{2} x_{2k} + \pi = \frac{2}{3}\pi + k \cdot 2\pi$

$\Rightarrow x_{2k} = \left(\frac{2}{3}\pi - \pi + k \cdot 2\pi\right) \cdot \frac{2}{\pi} = \left(-\frac{1}{3}\pi + k \cdot 2\pi\right) \cdot \frac{2}{\pi}$

$\underline{x_{2k} = -\frac{2}{3} + 4k}$